On linear combinations of idempotent matrices

Jerzy K. Baksalary[1] **and Oskar Maria Baksalary**[2]

[1]*Zielona Góra University, Zielona Góra, Poland*
[2]*Adam Mickiewicz University, Poznań, Poland*

Abstract

Two problems considered recently in the literature, both dealing with linear combinations of idempotent matrices, are revisited. The first of them was posed by Coll and Thome (2003) and concerns the question of when a linear combination $\mathbf{T} = c_1\mathbf{P}_1 + c_2\mathbf{P}_2$ of nonzero different complex idempotent matrices $\mathbf{P}_1$, $\mathbf{P}_2$, with nonzero complex numbers c_1, c_2, is the group involutory matrix. According to the solution provided by Coll and Thome (2003) as Theorem 1, it is possible in a finite number of cases, each characterized by definite values of scalars c_1 and c_2. In the present paper it is shown that this theorem is not correct, for the actual number of cases in which $\mathbf{T}$ is the group involutory matrix is infinite.

The second problem was posed by Özdemir and Özban (2004) who considered the question of when a linear combination $\mathbf{P} = c_1\mathbf{P}_1 + c_2\mathbf{P}_2 + c_3\mathbf{P}_3$ of nonzero different, mutually commuting, complex idempotent matrices $\mathbf{P}_1$, $\mathbf{P}_2$, $\mathbf{P}_3$, with nonzero complex numbers c_1, c_2, c_3, satisfies $\mathbf{P}^2 = \mathbf{P}$. The answer to this question given in the above mentioned paper as Theorem 3.2 does not provide the complete solution to the problem, for it characterizes particular sets of sufficient conditions only. This lacuna is covered by the present paper, in which the list of necessary and sufficient conditions ensuring that $\mathbf{P}$ is an idempotent matrix is obtained. Parenthetically notice that this result generalizes part (a) of Theorem 1 in Baksalary (2004).

Keywords

Commutativity, Group inverse, Group involutory matrix, Idempotency, Oblique projector, Projector, Tripotent matrix.

References

Baksalary, O.M. (2004). Idempotency of linear combinations of three idempotent matrices, two of which are disjoint. *Linear Algebra Appl. 388*, 67–78.

Coll, C. and N. Thome (2003). Oblique projectors and group involutory matrices. *Appl. Math. Comput. 140*, 517-522 .

Özdemir, H. and A.Y. Özban (2004). On idempotency of linear combinations of idempotent matrices. *Appl. Math. Comput. 159*, 439-448 .