On the minimum rank for pattern matrices*

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Abstract

By a *partial matrix* we mean an $m \times n$ matrix $\mathbf{P}$ some of whose entries are specified and the remainder entries are free variables of some indicated set. A *completion matrix* of $\mathbf{P}$ is an specification of the free variables obtaining a conventional matrix. Matrix completion problems try to obtain conditions for the existence of a completion for a given partial matrix in a class of interest. The historical motivation for the study of these problems appears in subjects as mathematical economics, biology, social sciences, etc. in which models some matrix elements give qualitative instead of quantitative information. A *pattern matrix* $\mathbf{P}$ is a partial matrix with the specified entries equal to zero and the remainder entries are nonzero free variables over a field $\mathbb{F}$. We say *pattern*$(\mathbf{A}) = \mathbf{P}$ if $\mathbf{A}$ is a completion matrix of $\mathbf{P}$. An interesting matrix completion problem asks for those completions for a given pattern matrix $\mathbf{P}$ with the lowest possible rank. By $\mathrm{mr}(\mathbf{A})$ we denote the *minimum rank* of $\mathbf{P}$ that is $\mathrm{mr}(\mathbf{P}) = \min\{\mathrm{rank}(\mathbf{A}) | \mathrm{pattern}(\mathbf{A}) = \mathbf{P}\}$. In this work we consider a graph theoretic approach to study the minimum rank completion problem and obtain different lower and upper bounds for solving the question.

Keywords

Pattern matrix, Minimum rank, Bipartite graph.

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