Projection methods for the linear split feasibility problems

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Abstract

Let $C \subset \mathbb{R}^n$ be a closed convex subset, $\mathbf{A}$ - an $n \times m$ real matrix and $\mathbf{b} \in \mathbb{R}^{\mathbf{m}}$. Consider the following linear split feasibility problem (LSFP)

$$\text{find } \mathbf{x} \in C$$

$$\text{such that } \mathbf{A}^T\mathbf{x} \leq \mathbf{b},$$

if such $\mathbf{x}$ exist. The problem has lot of applications, e.g., the problem of computed tomography or the problem of intensity modulated radiation therapy can be modelled as a large scale LSFP. We study a projection methods for the LSFP which generate a sequence $(\mathbf{x}_k)$ by the following iterative scheme

$$\mathbf{x}_{k+1} = T(\mathbf{x}_k), \tag{1}$$

where $\mathbf{x}_1 \in C$ is arbitrary, the operator $T : C \longrightarrow C$ is defined by the equality

$$T(\mathbf{x}) = P_C(\mathbf{x} - \mu\gamma(\mathbf{x})Gw(\mathbf{x})), \tag{2}$$

$\mu \in [0, 2]$, is called a *relaxation parameter*, $\gamma : C \longrightarrow \mathbb{R}_+$ is called a *step size function* and $w : C \longrightarrow \mathbb{R}^m_+$ is called a *weight operator*. We call the method (1) with T defined by (2) the *projected surrogate constraints method* (PSC-method). We study in the paper the Fejér monotonicity and the convergence of the PSC-method in dependence of step size γ and weights w. We show also that the convergence of the surrogate constraints method of Yang-Murty and of the CQ-method of Byrne applied to the LSFP follows from our main result.

Keywords

Linear split feasibility problem, Projection methods.

References

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