Matrix comparisons via eigenvector majorization: some results

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Abstract

We discuss some aspect of matrix comparison w.r.t some class of criteria. Consider a set $\mathcal{M}$ of (t,t) symmetric matrices, we define a criteria to be a map from $\mathcal{M}$ onto $(-\infty, +\infty)$. A matrix M is minimal w.r.t. ϕ if it minimizes $\phi(M)$ over $\mathcal{M}$. Such kind of problem occurs typically in experimental designs, where $\mathcal{M}$ is a set of information matrices and ϕ is usually a convex, non-increasing function. We are interesting particulary in the Φ_p criteria defined by:

$$\begin{array}{rcl} \Phi_p(M) & = & = \left(\frac{1}{t-1}\, tr(M^{-p})\right)^{1/p}, \text{ for } p \neq 0 \\ \Phi_0(M) & = & \lim\limits_{p\to 0} \Phi_p(M) = \det(M)^{-1/t}, \end{array}$$

We present some sufficient condition for a matrix to minimize simultaneously all the Φ_p criteria for p greater than some p_o. For example, we have:

$$\lambda(M_{d_1}^{-p_o}) \prec \lambda(M_{d_2}^{-p_o}) \Longrightarrow \left\{ \begin{array}{rcll} \Phi_{p_o}(M_{d_1}) & = & \Phi_{p_o}(M_{d_2}) & \\ \Phi_p(M_{d_1}) & \leq & \Phi_p(M_{d_2}) & \text{for } p > p_o \\ \Phi_p(M_{d_1}) & \geq & \Phi_p(M_{d_2}) & \text{for } p < p_o \end{array} \right. ,$$

where $\prec$ is the majorization ordering (see Marshall & Olkin, 1975). Note that the converse part does not hold. We also present similar results, some of them using weak majorization and we give some applications

Keywords

Majorization, Schur convexity, Matrix comparison.

References

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