

G-majorization and matrix inequalities

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Abstract

A unified approach is presented to the vector inequalities of the form $Lx \preceq Kx_{\downarrow}$, where L and K are linear operators, \preceq is a group induced cone ordering and $(\cdot)_{\downarrow}$ is the normal map associated with the ordering. In particular, a G-majorization inequality involving two orthoprojectors is given. The inequalities generalize a variety of majorization results on eigenvalues and singular values of matrices. The results are interpreted for various classes of matrices.

Keywords G-majorization, group induced cone ordering, singular value, eigenvalue, doubly graded matrix, tournament matrix, absolutely doubly substochastic matrix, Hadamard product.

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