The conjugate gradients methods with indefinite preconditioning

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Abstract

In this contribution we consider the solution of symmetric indefinite linear systems of the form

$$\begin{bmatrix} A & B \\ B^T & 0 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} f \\ g \end{bmatrix},$$

where A is a symmetric positive definite matrix and B has full column rank. Saddle–point problems of this type arise in many application areas such as computational fluid dynamics, electromagnetism, optimization and nonlinear programming. Particular attention has been paid to the iterative solution of these systems and to preconditioning techniques. Several structure–dependent schemes have been proposed and analyzed. Indeed, the block pattern of saddle-point systems enables to take into account not only simple preconditioning strategies and scalings, but also preconditioners with a particular block structure. Here we analyze the null-space projection (or constraint) indefinite preconditioner. Since it was shown that the behavior of most of nonsymmetric Krylov subspace methods can be in this case related to the convergence of preconditioned conjugate gradient method (PCG) we study in detail its theoretical properties and propose simple procedures for correcting its possible misconvergence. The numerical behavior of the scheme is discussed and the maximum attainable accuracy of the approximate solution computed in finite precision arithmetic is estimated.

This contribution is a joint work with V. Simoncini and it is partially supported by the project 1ET400300415 within the National Program of Research "Information Society".

References

Rozložník, M. and V. Simoncini (2002). Krylov subspace methods for saddle point problems with indefinite preconditioning. *SIAM J. Matrix. Anal. Appl. 24*, 368–391.