Structured matrices and efficient algorithms of polynomial interpolation

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Abstract

The problem of constructing efficient algorithms of polynomial interpolation involves matrices with special structure. Numerical algorithms should exploit the structure of such matrices. Horner's, Goertzel's, Clenshaw's methods and the scheme and the scheme of divided differences are frequently used in the interpolation and approximation problems and in signal processing. In order to improve accuracy we propose modifications of these algorithms. A comparison of various techniques with respect to efficiency, numerical stability and accuracy is given.

The numerical tests in Matlab demonstrate the computational advantages of the proposed modifications.

Keywords

Structured matrices, Polynomial evaluation, Interpolation, Clenshaw's method, Goertzel's algorithm, FFT, Condition number, Numerical stability, Iterative refinement.

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