MINRES residual norms of diagonally translated linear systems

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Abstract

Symmetric indefinite linear systems are currently solved by either preconditioned direct [2, 3, 4] or preconditioned iterative [1, 6] methods. Among the methods that are being preconditioned, the MINRES method [10] belongs to the most popular ones. As a well-known matter of fact, convergence behavior of MINRES can be related to the eigenvalues of the involved system matrix. Hence a diagonal translation, shifting the spectrum, can have an important influence on the convergence speed of MINRES processes.

In this talk we compare the MINRES residual norms obtained from two linear systems whose matrices differ by a diagonal translation. The equation we derive does not refer to spectral properties of the matrix, but it is based on an exact expression for the residual norm of residual minimizing methods such as MINRES [7]. The comparison is a generalization of the relation between the convergence for systems with translated matrices that has been formulated in [8, 9] for the special case of tridiagonal Toeplitz matrices. Surprisingly, the residual norms appear to be mutually related in a rather complicated way [5].

The result was obtained in joint work with Zdeněk Strakoš.

Keywords

Submission of abstract, Reference style, Submission date, How to submit.

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